The development of noticing in primary school mathematics teachers[1]

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ABSTRACT

The need to grow mathematics teacher practices and improve learning in classrooms is an important avenue for research. The aim of this article is to present results of a Lesson Study-based professional development programme that endeavoured to capture and understand the concept of teacher-noticing in two South African primary school mathematics teachers. Since teacher-noticing is a precursor to teacher decision-making, the question of how mathematics teacher-noticing develops was the focus of this study. A professional development programme set within teachers' own classrooms (where they played the role of noticers) was designed and implemented. Two Grade 6 mathematics teachers volunteered to take part in the study. The researcher and participant teachers collaboratively planned lessons that were taught by the researcher. Teachers took notes of their noticing during each of the four researcher-taught lessons to discuss during the reflective sessions after the lessons. These sessions were recorded and transcribed. The transcriptions were coded for emerging and developing themes in teacher-noticing using a teacher-noticing framework. It was found that teacher-noticing remained at lower levels during traditional direct instruction lessons while teachers developed extended noticing from lessons that were structured and planned along a problem-centred or modelling approach and that involved extensive pair work.

Keywords: mathematics teacher development, noticing, lesson study, primary school mathematics

INTRODUCTION

Mathematics teacher development has been a concern for a number of years (Simon & Schifter, 1991; Evan & Ball, 2009; Artzt et al., 2015). The need to grow teacher practices and improve teaching and learning in classrooms is an important avenue for research in mathematics education. Understanding how teachers make day-to-day decisions in their classrooms needs to be considered when planning teacher professional development (TPD) programmes. Since 'teachers' noticing is intimately tied to their orientations (including beliefs) and resources (including knowledge)' (Schoenfeld 2011: 231), it is a complex and integrated teaching competency. What teachers notice will influence their decisions, which in turn may affect their classroom practices. Roth McDuffe et al. (2018: 175) surmised that noticing 'involves teachers' attention to classroom actions and interactions, reflections, reasoning, decision and actions,' and further concluded that noticing includes what teachers see, how they make sense of what

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they see and their subsequent decisions. In other words, teachers see with their minds as much as with their eyes (Scheiner, 2016). However, teacher-noticing is not a means to its own end, but results in responsive teaching (Gibson & Ross, 2016).

LITERATURE OVERVIEW

On the surface, it may appear that we have no control over what we notice. We notice things that are more conspicuous than others. A loud noise, a bright sign or sudden movement. We are bombarded by a continual flow of cues from our environment. However, during teaching, noticing is not an incidental activity, but a professional one (Ball, 2011). Noticing in the domain of classrooms is more complex. To notice more or to develop the sensitivity to notice requires effort and is something we intentionally decide to do (Mason, 2002). Teacher-noticing is how teachers make sense of the vast amount of sensory data that a classroom brings with it and is therefore not a passive process (Sherin, Jacobs & Phillips, 2011). These authors also set out that since teachers are active in the classroom setting, teacher-noticing involves two processes: (i) attending to particular events in the setting (what does a teacher attend to?) and (ii) making sense of those events (abstracting these events).

Dreher and Kunste (2015) confirm Schoenfeld's (2011) view that teachers notice things based on their professional knowledge and views. It may therefore be conjectured that teachers who have deeper subject-matter knowledge and pedagogical-content knowledge (Hill, Ball & Schilling, 2008) may be better noticers. Star and Strickland (2008) point out that the ability to learn from teaching is dependent on the ability to notice. If this is the case, then teacher-noticing is a fundamental aspect of understanding teacher learning in TPD. It is anticipated that through a Lesson Study (LS) environment, teacher-noticing can be developed and honed so that meaningful teacher development can take place. LS is a TPD environment where a group of teachers and specialists or researchers plan lessons collaboratively. The lesson is then presented by one member of the group while the others are active observers, often looking for links between teaching and learning or how learners learn mathematics. The lesson is followed up by a reflection session to discuss the lesson and to refine planning for the next lesson.

Star and Strickland (2008) found in a study on pre-service teacher-noticing of video lessons that pre-service teachers scored high on noticing classroom management issues while their lowest score was on noticing the classroom environment and the mathematical content of the lesson. In a noticing-via-video analysis study by Mitchell and Marin (2015), a framework analysis was used to guide teacher-noticing through videos by focusing on important issues in a mathematics classroom. However, Mitchell and Marin considered their Mathematical Quality of Instruction framework limited in that it did not consider the substance of student engagement in a classroom and also did not reflect on the depth of student reasoning. Sherin and Van Es's (2005) study on teacher-noticing found that teachers changed what they noticed (from teacher pedagogy to student thinking) and the way in which they discussed what they had noticed (from evaluation to interpretation and increased use of evidence-based comments). These teachers took videos of their own classes to discuss at monthly meetings but were not given a specific framework to consider when noticing in classrooms because the researchers wanted to leave the noticing open to teachers. This idea is enacted in the study reported on in this article in order to keep noticing open and authentic.

Van Es (2011) points out that learning a new discourse is central to teacher-noticing and that a noticing discourse goes beyond describing or evaluating an event. Noticing requires an interpretive discourse. It is unlikely that teachers will produce this type of discourse on their own. This study proposes that the LS process can facilitate discourse changes when teachers reflect together on their noticing of a lesson in post-lesson interviews. Researchers (as part of the LS group) can extend the noticing of teacher participants by sharing what scholars in the field have found with the teachers.

How do various scholars gauge teacher-noticing? Table 1 integrates a number of frameworks that exist in the literature on teacher-noticing. For the purposes of this study, the researcher embedded various frameworks into one structure to harness the thinking of a number of scholars on teacher-noticing and to integrate what is already known about teacher-noticing frameworks. While scholars extended the thinking of previous scholars in terms of who is identified or what topic is discussed when teachers notice, this study focuses on the stance teachers take when noticing since the stance includes agents and topics. Van Es' (2011) levels (shaded in the table) are used when coding the data since it allows for coding across lessons and topics.

Table 1:
Integrated noticing frameworks

Overarching Framework (Van Es and Sherin (2006)		Extended framework(s)	
Agent (Who is identified)	• Teacher • Student • Other	• Student-Student • Student-Teacher • Student-Materials • Verbal and non-verbal (Mason, 2002)	
Topic (What is discussed)	• Mathematical thinking • Pedagogy • Climate • Classroom management	• Environment • Management • Tasks • Content • Communication (Star & Strickland, 2008)	• The mathematics • Cognitive demand • Equitable access to content • Agency, ownership and identity • Formative assessment (Schoenfeld, 2014)
Stance	• Description • Evaluation • Interpretation	• **Baseline (Level 1)** Mostly descriptive statements • **Mixed (Level 2)** Descriptive with some evaluative statements • **Focused (Level 3)** Provides evaluative comments • **Extended (Level 4)** Makes connections between teaching and learning Van Es (2011)	
Focus	• One or more categories • Narrow or broad		

Teachers must also be given the correct space and time *to notice*. Therefore, an LS TPD programme was designed so that teachers could notice in their own classes while the researcher taught the lesson. Since Mason (2011: 37) stipulates that noticing often happens retrospectively and not necessarily 'in the moment', the reflection phase of LS serves as a viable intervention design for a teacher-noticing TPD. It is in this reflection phase of LS that metacognitive self-observation may develop to enhance teachers' productive noticing while also enabling teachers to develop the inquiry stance needed to learn through LS (Choy, 2016). When teachers are allowed to spend time observing a lesson and then prompted to reflect on their observations, many metacognitive processes are promoted. It is through this reflective metacognition that enhanced teacher-noticing takes place (Leavy & Hourigan, 2016).

The study wanted teachers to be active observer-noticers in the lessons. The question of how to allow teachers to notice and capture this noticing needed consideration. In some studies on teacher-noticing, video clips of other teachers' lessons were used and the participants of these studies were asked what they noticed (Sherin, Russ & Colestock, 2011). Here, the participants had a common view and stance of someone else's lesson but this could differ from noticing in their own lessons. In other studies, teachers were asked to retrospectively recall noticing during their own lessons in either individual interviews or group sessions (Sherin & van Es, 2005). Another study involving new technologies, such as micro cameras, were used (Sherin, Russ et al., 2011) and teachers were asked to record something interesting that happened during their lessons to share and reflect upon. The idea of 'interesting' was left purposively vague by the researchers so that teacher-noticing was not specifically directed to any aspect. This was decided so that the researcher did not give teachers any particular 'examples' that they may consider to be correct. It would also enable the teachers to be more active observers rather than passive onlookers (Sherin & Star, 2011).

This study, however, adapts typical LS approaches (such as collaborative lesson planning and observing of lessons by teachers followed by collaborative reflection sessions), in that the researcher will teach the lessons while the participating teachers will observe and notice during the lessons. A researcher-taught lesson is congruent with what Clarke et al. (2013) and Bruce et al. (2009) termed a 'demonstration lesson'. This is a deliberate decision on behalf of the researcher to provide teachers with the space and time 'to notice' during the lessons. The lessons will take place in the participating teachers' own classrooms in order to bridge the divide that can form when professional development takes place away from teachers' classrooms. Teachers will also be involved in the planning of the lessons so that their lessons match their curriculum and classroom needs. This design, therefore, answered the following research question:

> How does teacher-noticing develop through a lesson study approach where teachers act as observer-noticers during mathematics lessons taught by the researcher?

The study aims to meet Sherin and Star's (2011) call to develop a more comprehensive model of teacher noticing by giving teachers an opportunity to notice and to reflect on their noticing without being hampered by presenting the lesson while being involved in the planning of the lesson. Furthermore, the lessons will be conducted in their own classrooms with their own learners which may allow for more meaningful noticing.

MATERIALS AND METHODS

The research can be considered part of the interpretivist qualitative paradigms. The strategy is that of LS's reflective collaborative post-lesson focus groups. The researcher-as-teacher in this study meets a number of needs. On the one hand, teachers are usually reluctant to 'be observed' while it is also difficult to learn how to 'notice' while teaching. The lessons were co-prepared with the participating teachers about a week before the lesson presentation took place. The two teachers informed the researcher of the topic and at what level it should be taught. They shared their previous lesson ideas and the textbook with the researcher. The first lesson was taught to specific teacher instruction (and included a specific method that the teachers wanted taught). In the second and third lessons, the researcher was given some guidelines, while in the fourth lesson, the researcher was given carte blanche on what and how to teach since the teachers had completed most of their curriculum and were doing revision.

Two Grade 6 teachers who taught mathematics at one school were purposively selected to take part in the study. Their willingness to take part was a major role-player in their selection as was the convenient proximity of the school to the researcher. Both teachers were experienced (over 10 years of mathematics teaching) and both were teaching at a well-resourced school. These teachers had interactive boards and laptops in their classrooms. The learners had their own textbooks and many resources were available in the classes (posters, equipment, games, etc.). The classes (of approximately 33 learners) were of mixed race, gender and ability.

During the four research lessons, both teachers acted as noticers. After the lessons, the teachers and the researcher held a reflective session. The teachers were given notebooks to jot down their noticing during the lesson and they referred to these notebooks during the discussions. The reflective discussions would allow teachers to take part in 'thinking aloud' (Swennen, Lunenberg & Korthagen, 2008). The reflective discussion with the researcher was audio recorded and transcribed. During these reflective discussions, the researcher started with 'What did you notice in the lesson?' Each teacher was given an opportunity to read her written notes. Towards the end of the discussion, the researcher also asked 'If you had to teach this lesson again, what would you do differently?' and 'What/How will you teach in the next lesson?' to prompt further noticing discussion.

During the first lesson, the researcher taught the lessons as specified by the teachers. Both the content, level and method for long division was given by the teachers and an idea of how they usually taught the lesson. For the second research lesson on capacity, the researcher designed a practical lesson. After a preliminary warm-up activity where pairs of learners had to match pictures of certain containers with a corresponding number capacity card (card-matching), learners worked in groups of four, where they measured the capacity of six different (unmarked) containers. The learners first had to look at all six containers and estimate the capacity before being given measuring jugs and buckets of water to measure the actual capacity of these containers. The lesson was designed so that participating teachers could notice learner thinking about capacity and measurement.

The third lesson was on number patterns and translating between flow diagrams, table patterns and algebraic rules. Learners worked in pairs on card-matching activities as in Figure 1.

Figure 1:
Lesson 3 card sorting and matching

Input	1	2	3	4	23	**x 2 + 1**	**47**
Output	3	5	7	9			
Input	1	2	3	4	18	**x 3 - 2**	**52**
Output	1	4	7	10			
Input	1	2	3	4	21	**x 5 + 3**	**108**
Output	8	13	18	23			
Input	1	2	3	4	30	**x 8 - 3**	**237**
Output	5	13	21	29			
Input	1	2	3	4	50	**x 6 + 4**	**304**
Output	10	16	22	28			
Input	1	2	3	4	201	**x 2 + 2**	**404**
Output	4	6	8	10			
Input	1	2	3	4	50	**x 4 - 1**	**199**
Output	3	7	11	15			

Input	1	2	3	4	30	**x 7 - 3**	**207**
Output	4	11	18	25			
Input	1	2	3	4	40	**x 10 - 9**	**391**
Output	1	11	21	31			
Input	1	2	3	4	70	**x 5 - 5**	**345**
Output	0	5	10	15			

For the fourth lesson, the researcher presented a model-eliciting problem (Lesh & Doerr, 2003) to the class. This type of problem requires that learners produce a model of how they solved the problem. The groups were given a list of 15 names and the best performance of each person in a 100m race, an 800m race and a long jump. Their task was to create three groups of five learners so that any group stood a chance to win an upcoming athletics event. Part of the mathematical work of the task involved converting metres to centimetres, ordering decimal numbers as well as realising that for a 100m race, the smaller number indicates a better (faster) learner and that for long jump the larger number indicates the better (further) jump. Learners were not given any other instructions.

Ethics clearance was issued by the researcher's institution for the study, while permission from the overseeing provincial department of education and school principal was also secured. Parents and learners signed consent/assent letters before the study started. Teachers were assured of confidentiality and that they were taking part voluntarily and could withdraw at any time.

In terms of research trustworthiness, the following is relevant. Credibility is enhanced by the addition of verbatim teacher words from the transcripts. Transferability and dependability are also increased since enough detail of the TPD and the context of the participants are provided so that they can be adapted and used in another setting. Using an existing framework to code the data allowed for less researcher bias in trying to address the confirmability of the study.

FINDINGS

Before the findings are presented, the Van Es (2011) framework from Table 1 is briefly presented and further detail is provided.

Levels of Teacher Noticing

- **Baseline (Level 1)**
  Mostly descriptive statements. These are statements where the teacher presents what he/she sees or hears (sensory information).
- **Mixed (Level 2)**
  Descriptive with some evaluative statements. In these statements the teacher, will present something he/she sees/hears and then give a determining evaluation, judgement, opinion or summing up.
- **Focused (Level 3)**
  Provides evaluative comments. The teacher will make mostly evaluative comments regarding what and how mathematics is taught or learnt. The teacher may provide some interpretative statements in terms of generalising mathematics teaching and learning.
- **Extended (Level 4)**
  Makes connections between teaching and learning. The teacher will provide links between teacher actions and learner actions. The teacher may see cause and effect in what he/she notices in the

classroom. The teacher will interpret what they see in terms of mathematics learning and be able to propose alternatives in teaching.

In the first lesson (where teachers suggested the format and specified a method), teacher-noticing was mostly descriptive with evaluative statements added to most of the descriptive statements such as in the following example:

Teacher 2: The actual example of the Lotto [descriptive] was good [evaluative]. The weaker kids, though, when you said divide 120 million into 4 [descriptive] they just switched off completely because the number was so big [evaluative].

In this lesson, the teachers specified the long division method to be taught (divide, multiply, subtract, bring down or DMSB). This method was taught in the previous grades and teachers felt that it led to fewer calculation errors once students knew it and remembered it. The researcher taught the method and then allocated cards with worked-out examples to pairs of students. The pairs had to find the errors in the calculations after which a whole-class discussion was held.

The instances of interpretative statements all revolved around the pedagogy of the teacher (the researcher) as in this instance:

Teacher 1: Well, for one thing, pair work does work because it gives the other child confidence and I think sometimes peers teach them better. What I found very interesting is with the pair work – it was the first time one of the weakest kids put up his hand.

The transcriptions showed mostly general impressions – so, Level 1 from the Van Es framework is relevant to their noticing. This is consistent with Sherin and Star's (2011: 68) suggestion that noticing can be rare or non-existent in 'highly routinized' teaching that may describe this lesson. What was evident in the transcripts was that other than two references to the 'method' to be used by learners, the teachers made no statements regarding learners' understanding or thinking about division. There were many statements regarding a sub-group of learners; namely, what teachers called 'weak' learners. Also evident from teachers' noticing that pair work 'does work' is the belief that it is not a successful method or it is not a method that they use often.

In the discussion after the second lesson, the teachers also focused strongly on the sub-group of weaker learners. Teachers made many descriptive and evaluative comments while a few interpretative comments were evident.

Teacher 2: Pairing a strong and a weak learner does make a difference [evaluating].

Teacher 1: But this time the weak ones chatted a bit more and were more involved in this [interpreting].

The teachers also noticed the difference that visual representations made in this lesson:

Teacher 1: It wasn't a sum that they had to do, they had a picture and had to see what fits with the picture that made a difference [evaluating and interpreting].

Teachers also noticed that the traditionally faster learners were reacting differently to this lesson:

Teacher 2: They were looking around, they started completely doubting what they had done and removed it and 'maybe not' and that was interesting to see [describing and evaluating].

Teacher 1 made a comment regarding the practical work that reflected something that surprised her:

Teacher 1: I realised just how badly they know capacity [evaluating].

The teachers also noticed that learners had problems with actual measuring activity in terms of the error or parallax (which learners were alerted to during the lesson), understanding the gradations on the measuring cylinder and general inaccurate measurements. The researcher asked if it is because they generally do much more written than practical work during mathematics lessons.

Teacher 2: Definitely, that is part of the problem.

Researcher: Are they weak in the actual skill of measuring and reading or do you think they do not understand capacity?

Teacher 2: No, I do not think they actually understand capacity. I do not think they understand how tiny something is, say 5ml, while 5l is huge. I think they are just looking at the number and they are forgetting what the actual ml and l mean [interpreting].

Teacher 2: I don't think they have done enough visual work [evaluating] in the earlier grades to see the difference between ml and l [interpreting].

When discussing how they would re-teach the lesson or how they would teach the next lesson, teachers suggested that they would do practical measurement but help learners interpret the interval lines on the measuring cylinders. This is seen as a statement where the teacher makes connections between learner thinking and teaching strategies.

Teacher 1: And actually show them it's a number line [the measuring cylinder].

For this second lesson, teachers were still mostly on Level 1 noticing, but there were instances of Level 2 (mixed) and Level 3 noticing at times. This can possibly be attributed to the different instructional style of the lesson and the use of practical work that allowed teachers to notice more than when learners simply followed correct methods. The nature of the topic can also lead to different levels of noticing: as Teacher 2 noticed above, it was not the numbers that caused problems but visualising the difference between *ml* and *l*.

During the post-lesson discussion of the third lesson, many more interpreting and connecting (Level 3 and Level 4) statements are evident. Improved noticing may have come about as a result from the lesson activities, teachers' MKT or their beliefs about mathematics, since noticing is 'intimately tied' to teacher beliefs and knowledge (Schoenfeld, 2011: 231). In addition, the underlying drivers of teacher noticing are intertwined.

Teacher 1 explained the link between her proposed teaching strategy and learning:

Teacher 1: I would also give them a flow diagram to show them why. The difference is plus 4 [on the flow diagram] but it doesn't show plus 4 [in the rule], it is times by 4. They have to see the link that the flow diagram rule is exactly the same – it's the part you don't see in a table. You want them to see that and write the rule below [connecting].

Teacher 2: Going further, it will be good to give the table [with input numbers] 1,2,3,4,10,15,20 and give them the rule and see, because some of them will still just go plus 4, plus 4, plus 4 all the way [connecting].

The teachers also started thinking about a more learner-centred pedagogy and how they could enact this in their classes:

Teacher 1: You could have done that without teaching them because then maybe they would have picked it up. If you pair flow diagrams with different tables, that would have been interesting. [interpreting and connecting].

Teacher 2: It's a difficult balance – how much they can figure out on their own.

Although the Van Es (2011) framework is useful for analysing teacher-noticing about student learning, it does not make provision for teachers noticing their own thinking and orientations. In this case, the teacher was commenting on what Tyminski (2010: 295) termed 'teacher lust'.

Teacher 1: I didn't have to help one person today, with the previous lesson, I had to help… I couldn't help myself (laughs) it's difficult not to help! That was interesting for me.

In this example, the teacher is aware of her own orientation towards teaching mathematics: that she must help students when they struggle. Noticing frameworks need to be more comprehensive and include teacher-noticing of their own beliefs and orientations towards teaching mathematics, since their beliefs affect what they notice (Van Es, 2011).

For the fourth lesson, where a model-eliciting problem was used (these problems were new to the teachers), both teachers started the post-lesson discussion by indicating that the learners were not used to these problems and were confused:

Teacher 2: In the beginning they were all a bit… you could see, they didn't know where to start, where to go, what to do, some of them not a hundred percent sure if they understood what they were supposed to be doing, then some of them just started [describing, evaluating].

Once again, teachers started the discussion sessions by describing and evaluating. In the absence of specific teaching activities, the teachers focused on the task instead; they were not yet sufficiently focusing on learners and their mathematical learning so their noticing does not reach the depth of Level 4. Teacher 1 suggested simplifying the problem, showing that she was interpreting the early challenges learners experienced with the problem and connected this to a possible change needed in the task itself.

Teacher 1: Maybe start with two events only. One term do a 2-event problem and next term a 3-event problem to see if they learnt some strategy. Perhaps have shot put and not two running events [interpreting and connecting].

She continued by suggesting that the problem was too difficult for the weaker learners. She suggested giving weaker learners a list with only five names and results instead of 15. Her comments in this case relate to the task and not to the teacher or the learner. Additionally, the teacher's beliefs about how mathematics should be taught is evident. Her idea is that the task should be broken into smaller, more manageable tasks. Teacher beliefs are a strong influencer of teacher decision-making and teacher-noticing (Schoenfeld, 2011). In understanding teacher-noticing as an active process, their focus on weak learners and on the task provides information regarding *where* the teacher is actively looking for information (the properties of the environment that the teacher focuses on) to guide their noticing (Jazby, 2016).

Teachers experienced another feature of modelling problems: it is not always the typically mathematically strong learner that does well (Lesh, Zawojewski & Carmona, 2003; Biccard, 2017).

> **Teacher 1:** For example, Sam (pseudonym) said this is the fastest runner and the other [learner] said it is this one. Then you could see that he thought the other learner was right because he is the cleverer kid.
>
> **Teacher 2:** And yet it was surprising that he [more able learner] did not pick up a strategy immediately. Yes, he battled [describing and evaluating].

One learner in the class used an intuitive standard deviation, which the teachers noticed as unusual but did not recognise:

> **Teacher 2:** In one group, one boy started adding them together and got an average? And then, he added a few at a time to get the same answer as the average that he had… it's the weirdest thing I have ever seen [describing and evaluating].

Learners are known to use their own informal methods and self-developed tools to solve the problem (Hamilton, 2007). The implication is that teachers' mathematical knowledge for teaching (Hill, Ball & Schilling, 2008) which includes PCK and SMK need to be more extensive when open problems are given to learners. PCK and SMK directly affect teacher noticing (Dick, 2017).

The teachers still focused on sub-groups (weaker learners or more able learners) in their discussion:

> **Teacher 1:** What they [weaker learners] really enjoyed about the problem is that there wasn't a right answer. The weak kids were really excited that they weren't going to get something wrong [describing, evaluating and interpreting].
>
> **Researcher:** They were relieved that they could use a calculator.
>
> **Teacher 2:** And then none of them used a calculator (laughter).

The transcripts show that describing and evaluating were the dominant forms of noticing, but also that when teachers did move towards interpreting and connecting (Levels 3 and 4), it arose from Level 1 and 2 discussions. The descriptive context appears to be a necessary condition for Levels 3 and 4 noticing. It is unlikely that teachers (even at higher levels of noticing experience and proficiency) will start with a Level 3 or 4 noticing statement without first embedding it in the context (Levels 1 and 2). What is important is that teachers need to be supported to move beyond Levels 1 or 2 noticing. Gibson and Ross (2016) suggest that deep content knowledge leads to expert noticing in teachers; this was evident in teachers not identifying an intuitive standard deviation. Although teachers may have considerable teaching experience, developing their content knowledge alongside their noticing competencies may be necessary.

Table 2 summarises the main elements of teacher-noticing through the development programme. The third column provides an instance of 'researcher noticing' which implies that the research follows a three-tiered research design (Koellner-Clark & Lesh, 2003): where students notice things in their mathematical activities (tier 1), teachers notice student thinking (tier 2) and researchers notice teacher-noticing (tier 3).

Table 2:
Summary of noticing development

	Agents identified	Main topic of teacher-noticing	Stance	Other Features
Lesson 1	Teacher and Learners (subgroup)	Weaker learners Cause and effect of teacher actions	Describing Evaluating Interpreting – limited [Level 2]	No discussion on learning of concepts of the lesson, e.g. division (Teacher PCK needs to be developed to enable deeper noticing) Teachers saw pair work as a viable instructional strategy No noticing related to the task/method specifically due to routine teaching
Lesson 2	Learners	Basic concepts of capacity are not well developed and understood (Content) Noticing that practical work is necessary in previous grades	Describing Evaluating Interpreting – improving [Level 2]	Teachers see practical work as a missing element of learner conceptual understanding
Lesson 3	Teacher Learners Task	Link between flow diagrams and table patterns Noticing pedagogical value that linking different representations builds conceptual understanding	Describing Evaluating Interpreting [Level 3]	Teachers could not fully explain how going up in fours results in a multiplicative pattern (Teacher SMK needs to be developed to enable deeper noticing)
Lesson 4	Task Learners (subgroup)	Different solution strategies used by to solve the problem Learner developed strategies Change task to accommodate weaker learners	Describing Evaluating Interpreting Connecting – emerging [Level 3-4]	Varied learner solutions for modelling problem led to deeper teacher-noticing Teachers wanting to simplify and reduce the cognitive load of the task.

The lesson design allowed for teacher-noticing of their own environment and for the complex and dynamic interactions between teachers' cognitive and contextual resources and their noticing (Scheiner, 2016) to become evident. Traditional direct instruction lesson (where a method is presented) did not result in in-depth teacher-noticing, while a model-eliciting lesson produced noticing that included connections between teaching and learning. However, teachers may have become more proficient at noticing by the fourth lesson. In lessons 3 and 4, teachers also noticed specific task features, but still only in terms of how that affected sub-groups of learners.

When teachers were questioned later about their focus on the weaker learners, they responded:

> **Teacher 1:** [as teachers] we pick up on mistakes quicker and we look at what they are doing wrong rather than what they are doing right. Mistakes give us information and help us teach.

Teacher 2: We focused on the weaker learner because we found that the stronger and middle group seemed to be able to work quite independently, but the weaker learners didn't seem to cope as much. But with some of the topics, we were shocked at how well they did cope. It was good to see that they could do something that was presented differently.

Teachers were therefore noticing both what they anticipated (weaker learners not coping or getting incorrect answers) and what was surprising.

CONCLUSION

This study looked at emerging teaching-noticing. It was limited to two Grade 6 mathematics teachers who focused on noticing during four research lessons across different topics. Noticing progressed from Levels 1 and 2 in the first lesson, through to Levels 1 to 4 in the last two lessons. For the lesson context to be fully integrated into the noticing discussion, noticing needs to progress from Levels 1 to 4 during each lesson. Supporting teachers to move beyond Levels 1 and 2 noticing is an important consideration in professional development.

The study is significant on a number of levels. Firstly, it is one of a few studies on teacher-noticing in South Africa and provides the qualitative detail needed in the field of teaching-noticing (Scheiner, 2016). Secondly, the design of the lesson study is novel in that the researcher took on the teaching role so that the teachers could focus their attention on 'noticing'. It is also through the design (including practical work and modelling problem) that teachers noticed the value of both in developing conceptual understanding. The design was therefore beneficial to advancing the discourse on noticing but also for teachers' own development. Thirdly, the results indicate that the teachers in this study focused their noticing primarily on sub-groups of learners (that is, weak learners) which indicates where the teachers are actively looking for their information and how this guides their decisions. Further research is necessary to determine why this was used as a filter and what other filters teachers prioritise.

It may be necessary *to teach* teachers specifically how to notice. Both these teachers commented that the given time to observe and notice was very valuable to their understanding of mathematics teaching and learning. It is, however, recommended that these lessons are held more frequently and across one topic for an extended time in keeping with the lesson study idea that a series of lessons is planned, presented and reflected on. Teachers need more development on how mathematical learning takes place within curriculum topics (e.g. division, decimals, etc.) to engage in deeper noticing. Teachers know what 'markers' to look out for to show that learners have reached a certain level of procedural competency but not necessarily how learners move between competencies or how they reach conceptual competency. Teachers' mathematical knowledge for teaching (both PCK and SMK) as well as their own beliefs and orientations about mathematics affect their noticing stance, and what they notice (or do not notice). Teaching teachers specifically about noticing or guiding them to specifically notice learners' thinking (and not teachers' actions) during the lesson may also have resulted in deeper noticing. Leaving 'noticing' intentionally vague may not be advancing teachers' noticing discourses. However, to ascertain what teachers notice (without preconceived researcher notions) as a baseline study provides a footing for further studies. It would be valuable to repeat the research and provide teachers with more explicit noticing frameworks before they observe their first lesson. It is also recommended that teachers focus on and track a small group of learners, and not an entire class, especially in studies that involve more than one lesson. This will enable the teacher to concentrate noticing on fewer learners and may result in deeper noticing. It is important that we understand how teachers make sense of classroom activities and interactions when we decide how to support teachers. This raises the issue of how the nature of the content and the topic taught and teachers' own content knowledge affects noticing, resulting in the need for further research.

REFERENCES

Artzt, A.F., Armour-Thomas, E., Curcio, F.R. & Gurl, T.J. (2015) *Becoming a Reflective Mathematics Teacher: A Guide for Observations and Self-Assessment.* New York: Routledge.

Ball, D.L. (2011) Preface. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds.) *Teacher noticing: seeing through teachers' eyes.* New York: Routledge, pp.xx- xxiv.

Biccard, P. (2017) Mathematical modelling competencies of lower achieving grade 7 learners. *Journal of Educational Studies* 16(1) pp.218-235.

Bruce, C. D., Ross. J., Flynn, T. & McPherson, R. (2009) Lesson study and demonstration classrooms: Examining the effects of two models of teacher professional development. http://www.tmerc.ca/digitalpapers/samples/WholeResearchStory.pdf (Accessed 21 November 2017).

Choy, B.H. (2016) Snapshots of mathematics teacher noticing during task design. *Mathematics Education Research Journal* 28 pp.421-440.

Clarke, D., Roche, A., Wilkie, K., Wright, V., Brown, J., Downton, A., Horne, M., Knight, R., McDonough, A., Sexton, M. & Worrall, C. (2013) Demonstration lessons in mathematics education: teachers' observations foci and intended changes in practice. *Mathematics Education Research Journal* 25 pp.207-230.

Dick L.K. (2017) Investigating the Relationship Between Professional Noticing and Specialized Content Knowledge. In E. Schack, M. Fisher & J. Wilhelm (Eds.) *Teacher Noticing: Bridging and Broadening Perspectives, Contexts, and Frameworks. Research in Mathematics Education*. Springer, Cham.

Dreher, H. & S. Kunste. (2015) Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. *Educational Studies in Mathematics* 88 pp.89-114.

Evan, R. & Ball, D.H. (2009) *The Professional development and Education of Teachers of Mathematics: The 15th ICME study*. New York: Springer.

Gibson, S.A. & Ross, P. (2016) Teachers' professional noticing. *Theory into Practice* 55 pp.180-188.

Hamilton, E. (2007) What changes are occurring in the king of problem-solving situations where mathematical thinking is needed beyond school? In R.A. Lesh, E. Hamilton & J.J. Kaput (Eds.) *Foundations for the future in mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates Publishers, pp.1-6.

Hill, H.C., Ball, D.L. & Schilling, S.G. (2008) Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic specific knowledge of students. *Journal for Research in Mathematics Education* 39(4) pp.372-400.

Jazby, D. (2016) An ecological analysis of mathematics teachers' noticing. In B. White, M. Chinnappan & S. Trenholm (Eds.) *Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia)*. Adelaide: MERGA, pp.360-367.

Koellner-Clark, K. & Lesh, R. (2003) A modelling approach to describe teacher knowledge. In R. Lesh & H.M. Doerr (Eds). *Beyond Constructivism: Models and Modelling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Mahwah, New Jersey: Lawrence Erlbaum Associates Publisher, pp.159-173.

Leavy, A.M. & Hourigan, M. (2016) Using lesson study to support knowledge development in initial teacher education: Insights from early number classrooms. *Teaching and Teacher Education 57* pp.161-175.

Lesh, R. & Doerr, H.M. (Eds.) (2003) *Beyond Constructivism: Models and Modelling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Mahwah, New Jersey: Lawrence Erlbaum Associates Publisher.

Lesh, R., Zawojewski, J.S. & Carmona, G. (2003) What mathematical abilities are needed for success beyond school in a technology-based age of information? In R. Lesh & H.M. Doerr, (Eds.) Beyond *Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning and Teaching*. Mahwah, New Jersey: Lawrence Erlbaum Associates, pp.205-222.

Mason, J. (2002) *Researching your own practice: the discipline of noticing*. London: Routledge-Falmer Press.

Mason, J. (2011) Noticing: roots and branches. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds). *Teacher noticing: seeing through teachers' eyes*. New York: Routledge, pp.35-50.

Mitchell, R.N. & Marin, K.A. (2015) Examining the use of a structured analysis framework to support prospective teacher noticing. *Journal of Mathematics Teacher Education* 18 pp.551-575.

Roth McDuff, A., Choppin, J., Drake, C. & Davis, J. (2018) Middle school mathematics teachers' orientations and noticing of features of mathematics curriculum materials. *International Journal of Educational Research* 92 pp.173-187.

Scheiner, T. (2016) Teacher noticing: enlightening or blinding? *ZDM Mathematics Education* 48 pp.227-238. https://doi.org/10.1007/s11858-016-0771-2

Schoenfeld, A.H. (2011) *How we think: a theory of goal-orientated decision making and its educational applications*. New York: Routledge.

Schoenfeld, A.H. (2014) What makes for powerful classrooms and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Education Researcher* 43(8) pp.404-412.

Sherin, M.G., Jacobs, V.R. & Philipp, R.A. (2011) Situating the study of teacher noticing. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds). *Teacher noticing: seeing through teachers' eyes*. New York: Routledge, pp.5-13.

Sherin, M.G., Russ, R.S. & Colestock, A.A. (2011) Accessing mathematics teachers' in-the-moment noticing. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds.) *Mathematics teacher noticing: Seeing through teachers' eyes*. New York and London: Routledge, pp.79-94.

Sherin, M.G. & Star, J.R. (2011) Reflections on the study of teacher noticing. In M.G. Sherin, V R. Jacobs & R. A. Philipp (Eds.) *Mathematics teacher noticing: seeing through teachers' eyes*. New York: Routledge, pp.66-78.

Sherin, M.G. & van Es, E.A. (2005) Using video to support teachers' ability to notice classroom interactions. *Journal of Technology and Teacher Education* 13(3) pp.475-491.

Simon, M.A. & Schifter, D. (1991) Towards a constructivist perspective: An intervention study of mathematics teacher development. *Educational Studies in Mathematics* 22 p.309. https://doi.org/10.1007/BF00369293.

Star, J.R. & Strickland, S.K. (2008) Learning to observe: using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education* 11 pp.107-125.

Swennen, A., Lunenberg, M. & Korthagen, F. (2008) Preach what you teach! Teacher educators and congruent teaching. *Teachers and Teaching* 14(5-6) pp.531-542. doi:10.1080/13540600802571387

Tyminski, A. (2010) Teacher lust: reconstructing the construct for mathematics instruction. *Journal of Mathematics Teacher Education* 13 pp.295-391. doi:10.1007/s10857-009-9135-y

Van Es, E.A. (2011) A framework for learning to notice student thinking. In M.G. Sherin, V.R. Jacobs & R.A. Philipp (Eds). *Teacher noticing: seeing through teachers' eyes.* New York: Routledge, pp.134-151.

Van Es, E.A. & Sherin, M.G. (2006) How different video club designs support teachers in "Learning to Notice". *Journal of Computing in Teacher Education* 22(4) pp.125-135.