

A framework for the development and improvement of computational thinking for high school learners using a programming language and learner management system¹

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ABSTRACT

Many educational departments are losing the battle against inefficient mathematics education. The Annual National Assessments (ANA) and World Economic Forum reports tell a story that performance is declining annually among learners in South Africa. The study was conducted among Grade 9 learners at a private high school in the Western Cape to establish a framework for computational thinking. The problem statement reads that it is not clear how high school learners' computational thinking (CT) may be enhanced or improved at a cognitive level of formal operations. The research question posed is, 'How can CT be enhanced, among high school learners, using a PL aligned to Action-Process-Object-Schema (APOS) theory?' The research methodology was based on an interpretivist philosophy. The ontological underpinning of the study is subjective and the epistemological stance accepts opinions of learners through written, spoken and visual attributed meanings. The axiology of the researcher is that of a practising educator in programming, a teaching and learning expert and a certified Java-Alice-Greenfoot instructor through Oracle. The research strategy was based on educational design research as a validation study through interventions. Findings show that CT at a cognitive level of formal operations can be enhanced among learners through Greenfoot PL with APOS theory as lens. The support and recognition of the headmaster or line manager towards those involved in programming language (PL) and learning management system (LMS) education, determine the success of the roll-out.

Keywords: APOS Theory, cognitive level of formal operations, computational thinking (CT), embodied cognition, learner management system (LMS), programming languages (PL)

1. INTRODUCTION

The underperformance of learners² in especially mathematics at high schools in South Africa (RSA) remains a challenge for government, policymakers and educators (Spaull, 2013; Reddy, 2014; Reddy et al., 2015; Voogt et al., 2015). Reddy et al. (2015) argue that a magnitude of challenges exists at all

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2 The statistics showed the decline in the overall grades of Grade 9 learners (Spaull, 2013; HSRC, 2014; DBE, 2015). According to Spaull (2013) and Reddy et al. (2015), the Literacy and Numeracy Assessment (LaNA) rolled out by the Trends in International Mathematics and Science Study (TIMSS) indicates that South African learners are underperforming and hold the last position out of a total of 143 countries (Reddy, 2014).

levels of the pedagogical space when attempting to provide an ideal learning environment for all learners. The authors further posit that poor discipline, violence and bullying at schools have a negative effect on learners' mathematics and science performance. According to Reddy et al. (2012), poor and middle-class learners in Grade 8 are a barometer to predict who will pass in Grade 12. Many mathematics teachers serve middle-class learners, as categorised by Reddy et al. (2012), through Mathematical Powerhouses (MPHs). MPHs change the landscape of pass rates at any school. This is true where the costs of these services are covered for learners who are fortunate enough to be accepted by these MPHs.

Teaching towards mathematical problem solving (MPS) stays a challenge according to Chirinda and Barmby (2018). This challenge exists due to many factors within South African society, such as large or overcrowded classes and learner mathematical beliefs (Moscucci, 2007). According to Chirinda and Barmby (2018: 122), mathematics is 'inexplicable and beyond understanding for the majority of learners' in the RSA.

The RSA Department of Basic Education (DBE) has embarked on a National Development Programme with a five-year strategic plan (2015 to 2020), but implementation remains complex. Successful implementation of MPS is a challenge, as highlighted by Voogt et al. (2015), Denning (2017) and Lockwood and Mooney (2017). Systemic tests in RSA were introduced in 2018 as an upgrade of existing testing instruments such as the original Annual National Assessments (ANAs) (Schäfer, 2018). Reflective abstraction and algorithms form an important basis for the MPS approach (Aho, 2012; Cetin & Dubinsky, 2017; Denning, 2017; Chirinda & Barmby, 2018). The Action-Process-Object-Schema (APOS) theory is well researched on how mathematics learning takes place (Arnon et al., 2014) and has a strong connection with CT embedded in reflective abstraction (Cetin & Dubinsky, 2017). According to Selby and Woollard (2014), CT points to an abstract methodology and mathematical thinking to abstract structures. This study explores the effect of a programming language (PL) and the APOS theory on promoting CT skills among learners at Piaget's (1965) cognitive level of formal operations.

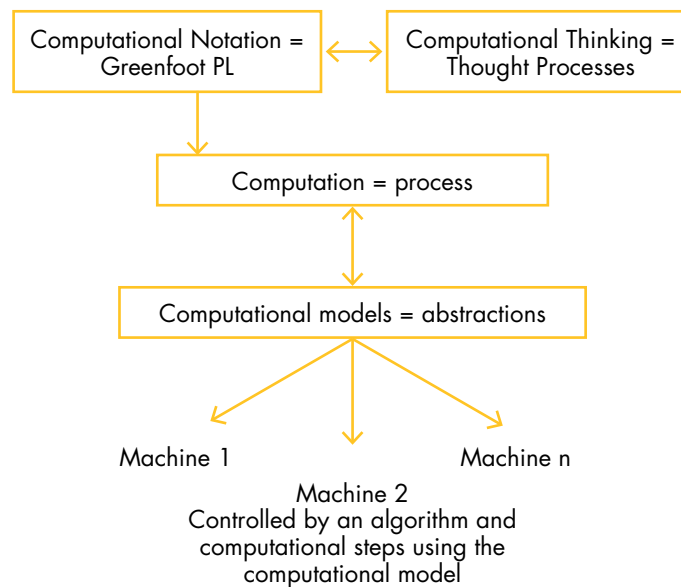
RSA educational departments made drastic changes to the current curriculum, which resulted in confusion among educationists as well as learners and parents (McCarthy & Oliphant, 2013). In an attempt to remedy the situation, the DBE provided textbook top-ups, mathematics and science kits as well as digital resources to be loaded onto notebooks to allow for a possible improvement in results in Grade 12 (Grant, 2012). According to Howie (1997, 2001, 2004, 2013), Howie and Plomp (2006), Maree et al. (2006), Reddy et al. (2006) and Reddy et al. (2015), these continuous curriculum changes post-1994 within the DBE seem to be an influential factor towards the status quo of mathematics and science results of learners.

The lack in CT may contribute to these negative trends as the curriculum does not provide adequate support for the development of CT at a cognitive level of formal operations. Interventions such as technology investments in schools without the implementation of CT as part of the rollout are a waste of resources and money (Pearson, 2008). The aims and work done by the DBE are commended and supported. This must be done against the background that the underlying problem is the underdevelopment of CT at the cognitive level of formal operations (Papert, 1980). Sharpening abstraction skills among Grade 9 learners may improve mathematical standards. The lack of CT among Grade 9 learners starts with the absence of abstraction skills. It becomes a challenge for educators to instil CT in learners. A possible solution can be found in the APOS theory which proposes the existence of mental structures when practising reflective abstraction (Arnon et al., 2014). APOS theory is a well-researched theory on mathematics learning (Dubinsky & Lewin, 1986; Dubinsky, 1991, 2000; Dubinsky & McDonald, 2001; Arnon et al., 2014). The discussion is highlighted in the following literature study section.

2. LITERATURE STUDY

Reflective abstraction is used in context of CT (Cetin & Dubinsky, 2017). Denning (2017) underlines Aho's definition (2012) of CT as the thought processes necessary to formulate problems. The solutions to these problems can be represented as computational steps and algorithms as depicted in Figure 1. Denning (2017) further argues that computation is a process consisting of a computational model together with computational thinking. An algorithm is a way to control any machine that uses the model. These computational models are abstractions at the core of computation and CT. Selby and Woollard (2014) researched the term 'CT' and found that three aspects are always found in the definition: (i) thought processes, (ii) the concept of 'abstraction' and (iii) the concept of 'decomposition'. They further argue that the terms 'problem solving' and 'logical thinking' are too broad and focus on skills development. Aho (2012) states that algorithms are implemented using a computational notation such as a PL.

Figure 1:
CT in motion (Compiled interpretation taken from Denning (2017) and Aho (2012))



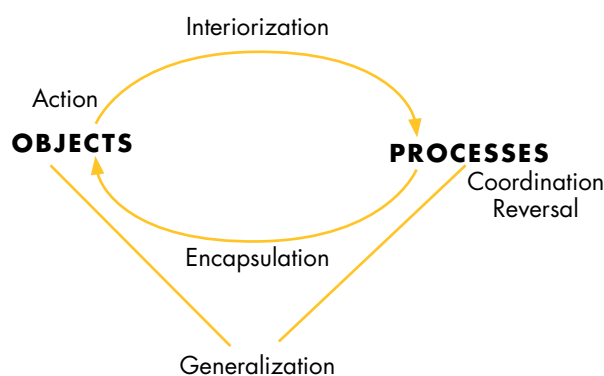
Abstraction and automation are the 'mental and metal tools' of CT (Wing, 2006, 2008: 3718). Words and phrases from 'thinking at multiple levels of abstraction', 'decomposition', 'heuristic reasoning to discover a solution', 'prefetching and caching in anticipation of future use', 'recursive thinking', and 'algorithm and precondition' describe some of the skills needed to think like a computer scientist (Wing, 2006: 33). Learners who master CT are able to understand a relationship among subjects and activities within and outside of school (Philbin et al., 2013).

Denning (2017) posits that following any sequence of steps or algorithm does not necessarily make you a computational thinker. Aho (2012: 2-3) states that CT is about finding appropriate models of computation to derive a solution for a formulated problem. Researchers such as Hayakawa (1949), Truran (1992), Wilensky (1991), Dubinsky (1991), Hazzan (1999, 2003), Devlin (2003), Kramer (2007), Perrenet (2009) and Meyer (2010) state subtle differences when arguing the concept of abstraction. Wilensky (1991: 4) states 'concreteness, then, is that property which measures the degree of our relatedness to the object, (the richness of our presentations, interactions, connections with the object), how close we are to it, if you will the quality of our relationship with the object'.

The APOS theory originated from Dubinsky's (1991) interpretation of Piaget's (1973) concept of reflective abstraction. Piaget sees the properties of objects not in the objects itself, but embedded in the actions

that learners take when using these objects (Arnon et al., 2014). Each mental construction (**A**ctions **P**rocesses **O**bjects **S**chemas depicted in Figure 2) or conception uses mental mechanisms (interiorisation, coordination, reversal, encapsulation and thematisation) to move through the APOS (mental structures) cycle. Reflective abstraction is a description of what goes on in the minds of individuals when engaged in creating knowledge. It is hypothetical as nobody can see what goes on inside another's mind (Dubinsky, 1991, 2000).

Figure 2:
Schemas and their construction (Dubinsky, 1991: 106)



There are many possible ways to solve a mathematical problem, which may confuse the learners' actions. This often leads to lower levels of abstraction, which complicates their understanding (Hazzan, 1999; Kramer, 2007). Researchers describe this state a learner is in as 'abstraction anxiety' (Sfard, 1991; Wilensky, 1991; Meyer, 2010), which forms an important component of mathematical anxiety. Papert (1980) uses the term 'mathophobia' and Tall (2004) refers to this phenomenon as 'dyscalculia'. Meyer (2010) states that educators should not only classify subjects as being abstract, but also deal with this anxiety associated with abstraction.

Many studies (Dubinsky, 1999, 2000; Hazzan, 1999; Wilensky, 1991; Dubinsky & McDonald, 2001; Kramer, 2007; Perrenet, 2009; Meyer, 2010; Maharaj, 2013; Brijlall & Maharaj, 2014) have been conducted on the concept of abstraction as a prerequisite for subjects such as Mathematics and Programming. However, many of these studies were conducted at university level where students had already acquired the skill of abstraction within certain disciplines. Instruments for assessing certain characteristics associated with abstraction skills are then devised or used to measure abstraction (Hill et al., 2008; Perrenet, 2009). Unfortunately, only a few studies have been conducted to date at school level from Grade 8 to Grade 12.

3. EDUCATIONAL DESIGN RESEARCH (EDR) METHODOLOGY

The EDR approach was followed with the emphasis on exploring and understanding CT at the cognitive level of formal operations. This was accomplished through teaching, learning and training Grade 9 learners (Bannan, 2013). A genetic decomposition (GD) through activities, classroom discussions and exercises (ACE) were used (Arnon et al., 2014). The GD is used as a model within the EDR framework, known as the Integrated Learning Design Framework (ILDF), proposed by Bannan (2013).

Plomp (2013) highlights three preliminary stages of EDR, influenced by many researchers. The stages are (i) preliminary research phase, (ii) prototyping phase and (iii) assessment phase. The interventions stated in Plomp's study were defined using the recommendations as output. This resulted in an iterative process

leading up to a possible solution, or in the context of this research, an educational practice to remedy the mathematical dilemma.

The Design for Intervention principle and the Greenfoot PL were used to enhance CT in Grade 9 learners using APOS theory as lens. Based on Van den Akker's (1999) questioning structures, research questions were revisited and combined to describe the technological educational intervention brought about by formative evaluation design principles. The research questions were adapted to fit the EDR methodology (Plomp, 2013) by asking the following question: What teaching and learning strategies can empower learners to master computational thinking skills in order to cope with the challenges in subjects such as Mathematics and Science?

4. THE DEVELOPMENT OF THE CT FRAMEWORK INCORPORATING PL AND LMS

For the development of the CT framework, formative evaluation is considered to be essential in ensuring high-quality interventions for this complex educational problem (Nieveen, 2013). The research focused on the following three phases, as identified by Nieveen (2013): (i) problem identification, (ii) orientation and positioning and (iii) implementation. All three stages are subject to formative evaluation.

4.1 *The problem identification phase*

This phase requires preliminary research that entails a literature review, conceptual framework and the identification of a target group, which are all discussed in the following sections.

4.1.1 *Needs and context analysis*

Learner performance in mathematics is deteriorating in South Africa, as has been pointed out in the introduction. Although there is a collective calling from the DBE, MPHs are making money from this mathematical dilemma South Africa is facing. The DBE attempts to enhance mathematical performance during a learner's final year of study at school, which raises questions such as, 'What was done in the former years of these learners towards their final year at school', and 'What is being done at this stage'?

The literature review of the study focuses on the typical mathematical approach, the discovery of CT, and the importance of understanding what is meant by abstraction and automation encompassing CT. The CT research conducted in this study concentrated on aligning the learner's understanding of APOS theory and how APOS can be embedded in a learner's mind without necessarily focusing on mathematics. Previous APOS research focused on university-level students. Inadequate APOS approaches among students have been pointed out in these studies, but only in conjunction with specific mathematical concepts and without real action taken other than an indication that students are not at a desired level of APOS (Maharaj, 2013). The poor Grade 12 results and the actual guidance for teachers and learners on how to intervene and salvage their mathematics approach or similar CT subjects form the foundation of this EDR.

4.1.2 *Literature review*

CT is a driving force for any learner when studying mathematics (Wing, 2008). Complex thoughts in the logic of individuals take place at a cognitive level of formal operations, but vary in complexity levels within the mind of the learner (Piaget, 1965).

Learners and teachers are not knowledgeable about the APOS theory as it is not included in the mathematics and science curricula of Grade 12 learners. The general approach to improve the high failure rate is to make use of concept images, but the images selected contribute to misconceptions instead of a better understanding of mathematical concepts. The objects and schemas used are disjointed and inevitably lead to poor results in mathematics (South African Mathematics Foundation, 2018).

4.1.3 Theory development

A theory, a framework and a model are needed to implement a practical approach with CT as outcome. The way learners interact with mathematics is a concern as it does not deliver positive outcomes (Spaull, 2013). The outcome of the literature review shows that the mathematical approach in schools is governed by working through papers and examples, which may strengthen a concept-image approach outcome (Reddy et al., 2012; Spaull, 2013; CDE, 2014; Reddy et al., 2015). The approach may also reiterate incorrect understanding of mathematical concepts because, according to Higgins and Wiest (2006), practice does not necessarily make perfect. Higgins and Wiest (2006) conclude that the learner may, by practising a relative or warped solution, create a conceptual image of a mathematical concept through a relative epistemological approach. Using a constructivist approach might force the learner to explore the mathematical problem without a guided or blended approach giving rise to a warped or relative idea of any mathematical solution. This may impact the epistemological views on mathematical education. It is argued that worked examples are important to minimise time spent on concepts. A worked example (Clark et al., 2010, 2012; McPhail, 2016) usually originates from the professional who has the right answer; it is not based on a relative viewpoint of the truth from a learner's perspective. Mathematical solutions to a mathematical problem may seem correct until the answer proves differently and is thus relative when seen from the learner's frame of mind. Solving a problem using a PL such as Greenfoot offers a trusted solution, which is underpinned by the APOS theory and immediately visible to the student. The LMS may provide a guided approach towards CT when solving a problem.

4.1.4 Target group

The majority of Grade 9 learners are faced with subject choices for Grade 10 that will affect their future. In Grade 9, external influences such as teachers and parents bring a more adult approach into the classroom. The learner has also grown one year in personality and attitude, and it is difficult to isolate one specific criterion affecting his/her viewpoint about learning. The complex multitude of factors (personality, attitude, etc.) that may play a role are ignored, as pointed out by Cegielski and Hall (2006). Grade 9 learners were therefore a logical choice as unit of analysis for this study.

4.2 The orientation and positioning phase

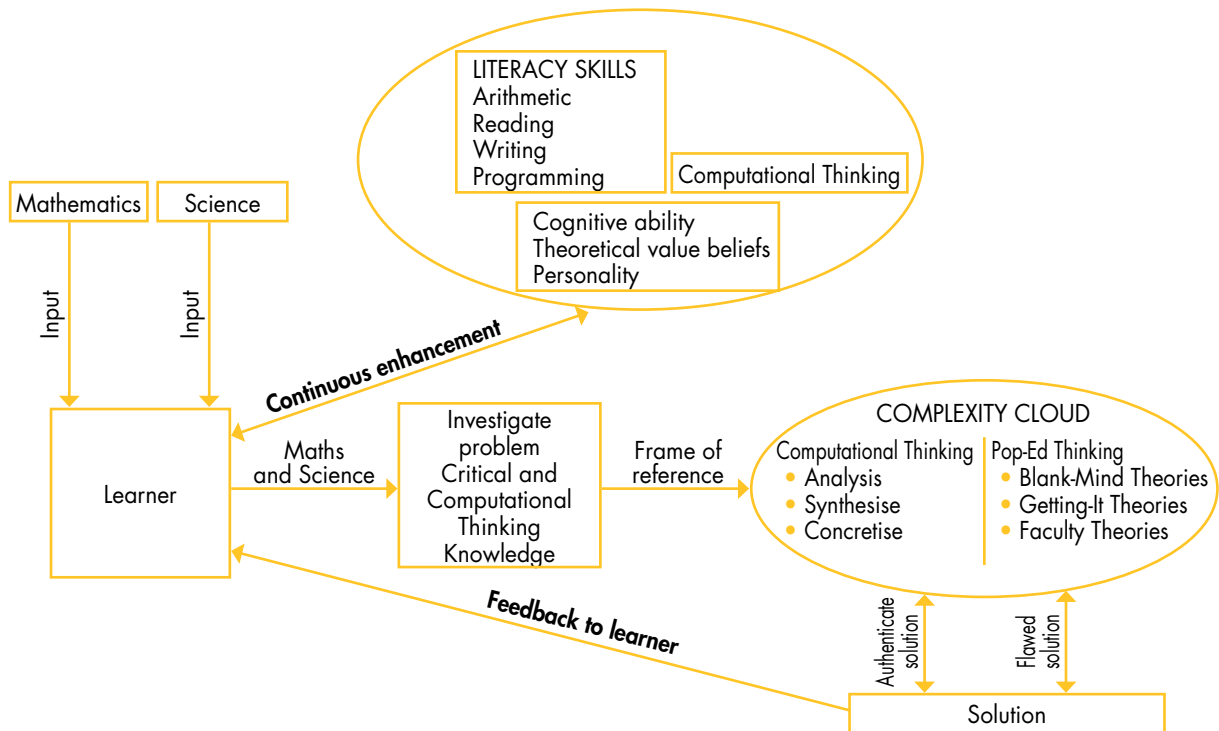
The orientation and positioning phase is the identification of the intervention and support of such interventions. According to Wademan, (2005), this is accomplished by adding tentative products and design principles to the intervention through formative evaluation. The next section deals with the conceptual framework and presentation mode.

4.2.1 Conceptual framework

Enactment, according to Bannan (2013), involves learning targets, innovation, choosing design principles, identifying and operationalising cognitive and performance processes in design, and how the design covers the theoretical model. Nieveen (2013) views the conceptual framework as the foundation of the intervention, based on the consideration of the curricular spider web by van den Akker (2003). The APOS framework was studied as the foundation of mathematics learning. Greenfoot PL houses many educational principles enhancing all of the APOS mental structures using mental mechanisms for each mental structure. The aim of a conceptual framework, designed by the authors and depicted in Figures 3, 4 and 5, may enhance CT among learners. This is achieved by transferring APOS mental structures to learners via the Greenfoot PL.

A basic theoretical conceptual framework was developed for the improvement of CT skills of learners (Figure 3). The interaction of learners and CT subjects such as Mathematics and Science are shown in Figure 3.

Figure 3:
The theoretical conceptual framework for the improvement of CT among learners in Grade 9



The learner investigates the problem from within his/her knowledge base by means of knowledge as well as CT. The frame of reference lies within what is called the 'complexity cloud'. The complexity cloud is based on research conducted by Bachelard (1938) who argued that existing cognitive beliefs may impact the progress of science, applied by Moscucci (2007) with her Mathematical Belief System Activity (MBSA). Papert (2005) also argues that thinking processes and beliefs may become obstacles for understanding mathematical concepts. Brousseau (1983) argues that certain existing knowledge may prevent the acquisition of new knowledge, also known as an epistemological obstacle. McGowen and Tall (2010) use the term 'met-before' which may impede further learning. This also holds true for the didactical situation (Brousseau, 2002) in class at the sample school. A number of aspects are hidden, and therefore not visible to the teacher, within the complexity cloud, such as the culture of Pop-Ed thinking (Papert, 1980) and CT (Perrenet, 2009). According to Papert (1980), Pop-Ed thinking consists of *blank-mind* theories, *getting-it* theories and *faculty* theories when learners are conditioned to believe that they are inferior and not able to solve problems or participate with other learners academically (Papert, 1980: 355).

This can give rise to a flawed solution (Jankvist & Niss, 2018) to the problem under investigation, as a learner is governed by theories that prevent him/her from developing a mathematical understanding of mathematical concepts as per definition. A typical example, depicted in Figure 4, was found among some of the learners within the study. They did not know the sum of the inner angles of a triangle. This could have been solved through an Action (first phase of APOS) given to learners: instructing them to cut off the corners of the triangle and place them adjacent to a ruler to illustrate 180 degrees.

Figure 4:

A learner's answer illustrating the lack of achieving the Action (first) phase within APOS theory

- f. Draw a triangle as perfect as can be.



- g. What is the sum of that triangle's angles in degrees?

unknown at this stage.

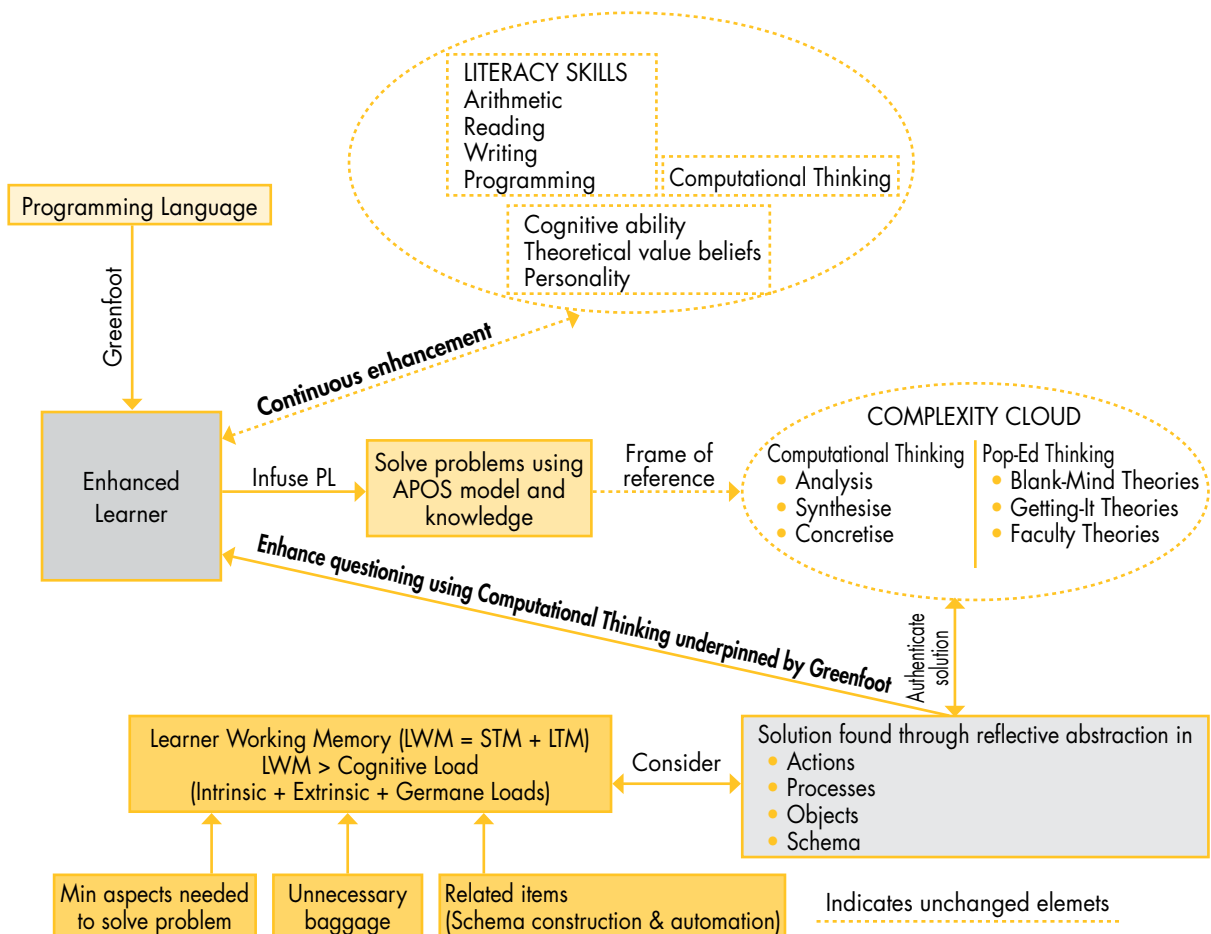
Not being at the Action phase prohibits the learner from advancing to the Process phase. According to Jankvist and Niss (2018) and Moscucci (2007), it is all about the learners' mathematical beliefs. These mathematical beliefs are made up of emotions, attitudes and cognition, also influenced by teachers unfortunately.

CT, on the other hand, consists of the analysis, synthesis and concretisation of mathematical concepts, which seems to be suppressed by the culture of Pop-Ed thinking, as has been found among learners in the sample group. The probability of a correct result of the CT component interaction is likely to be an authentic solution to the problem or striving towards an authentic solution. The solution, whether flawed or authentic, goes back to the learner. The learner uses the flawed or authentic solution in scaffolding new mathematical concepts, building towards a schema.

The continuous enhancement needed for the development of the learner is supported by the factors of literacy as put forward by Prensky (2008). CT is suggested by Wing (2008) as an additional factor to be literate at a cognitive level of formal operations (Figure 5), governed by theoretical value beliefs and personality (Cegielski & Hall, 2006). When investigative skills within CT are required, the learner enters a complexity cloud filled with either positive or negative thoughts that may sidetrack CT as pointed out previously.

Considering the basic framework (Figure 3), an intervention was designed using Greenfoot as PL installed on all personal lab computers (PCs). The facilitation of Java made it easier for system operators to use the existing Java JDK, also necessary for a Greenfoot installation. The learners then received an electronic folder containing the scenarios of the Greenfoot PL. The intervention was done by adding a PL as shown in Figure 5. Instead of having a situation where the problem was investigated by the learner only, the problem was solved using APOS structures through Greenfoot.

Figure 5:
An improved theoretical conceptual framework for enhancing CT among learners in Grade 9



The basic framework was revised to broaden the authentic solution through the use of reflective abstraction (Dubinsky, 1991) including Actions, Processes, Objects and Schemas or APOS framework. Learner Working Memory (LWM), which consists of short term (ST) and long term (LT) memory, is optimally used to streamline learning. Kranch (2010) states that ($LWM=STM+LTM$) increases the cognitive load consisting of intrinsic, extrinsic and germane loads (Figure 5). The intrinsic load refers to the minimum aspects needed to solve a problem. Contrary to this, extrinsic load refers to unnecessary baggage carried by any learner. The use of cell phone interaction by learners is a typical distraction. Germane loads refer to related items, including schema construction and automation for the purpose of learning. Schema construction involves equilibration, which is part of Piaget's theory of cognitive development. Equilibration is a dynamic process that learners undergo during assimilation or accommodation of schema construction, called cognitive adaptation (Bormanaki & Khoshkal, 2017). The learner can now be classified as an 'enhanced' learner.

4.2.2 Presentation mode

The presentation mode (Nieveen, 2013) of the intervention to accomplish computational thinking was built around infusing the Greenfoot PL. Formative evaluation led to the introduction of an LMS to enhance the quality of the learning experience as guided, not as haphazard constructivist approaches. Moodle was used to house most definitions on APOS and provide learner-teacher interaction beyond the notion of the curriculum. Furthermore, flipped-classroom techniques as well as YouTube videos were stored on an in-house-developed and password-protected Moodle website, <http://wrru.co.za/moodle>. Explanations of abstraction and other related concepts were structured in Moodle as a guided CT approach. During an

information meeting with teachers at another public school, it was found that they were not at all familiar with an LMS or the rollout of e-Learning in general. A formative evaluation approach was taken for the private school by structuring Moodle with a storyboard approach (Nieveen, 2013). This made Moodle more interactive in terms of teaching and learning strategies. The static presentation of most LMSs acts as a repository. This poses a problem at many educational institutions, with no embedded CT guidance of subject matter. Figure 6 illustrates the format of the intervention which includes more theories (Imenda 2014). This makes the theoretical framework a conceptual framework.

Figure 6:
Conceptual framework for improving CT using APOS, PL and LMS

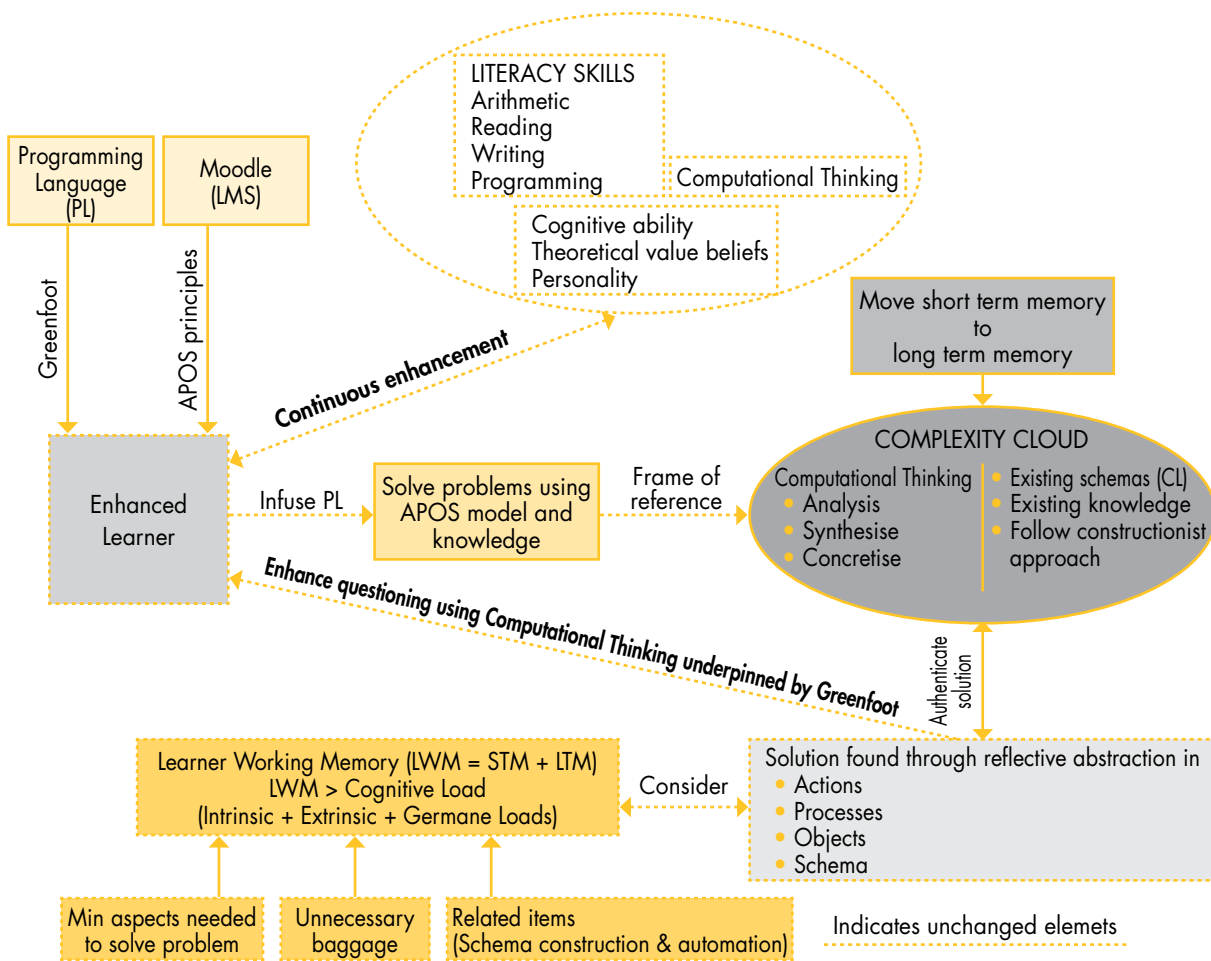
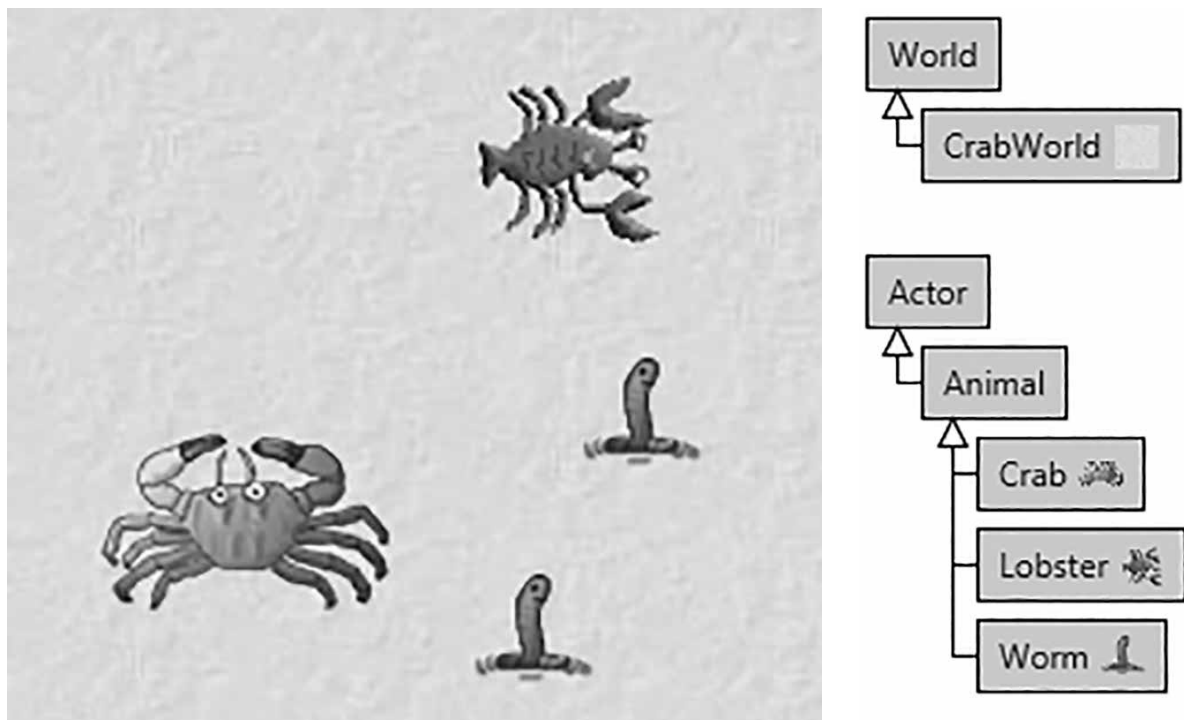


Figure 5 shows the introduction of a PL as the intervention. Using a PL created a challenge, as learners had to either rely on the internet or on the researcher’s explanations in class regarding usage of the environment or editor and aspects around many other syntax-related issues of a PL. Many learners were nervous when put on the spot to write a program due to insecure knowledge of fundamental aspects of a PL. Others were wary of peer pressure and therefore not able to complete a task. The intervention was therefore refined, as indicated in Figure 6, introducing an online LMS assistance, thereby enabling the learner to study programming language concepts using a guided approach. Moodle was made accessible to all learners in a ubiquitous manner, acting as a medium for constructivist learning in a controlled or guided fashion. The learner is guided with a correct epistemological approach (worked example). Initially, the learner had to construct his/her own knowledge based on a relative epistemological foundation (Figures 3 and 5).

The interactivity allows for performance measuring on certain exercises. Most important is the availability of a large collection of flipped-classroom examples referring to certain levels of programming in Greenfoot. Any learner may pre-study a 'worked example(s)' to enhance his/her learning, thereby not having to feel intellectually inferior. Within the complexity cloud of thought, pre-set ideas are replaced with a solid understanding of how CT should take place, as depicted in Figure 6. The 'enhanced' learner uses the PL and the LMS to solve problems intuitively by means of APOS mental structures using mental mechanisms. Furthermore, continuous enhancement is still happening and the questioning ability of the learner (CT inspired by the PL) is enhanced in order to solve problems using an APOS approach. For example, the learner creates a scenario in Greenfoot consisting of a world (CrabWorld) and the actors (Crab, Lobster, Worm), as depicted in Figure 7 below. Many scenarios such as this one can be downloaded by teachers from www.greenfoot.org/download if they need guidance on how to accomplish certain algorithms. During the research, we developed our own algorithms, but this scenario was used explicitly to help the reader to follow the explanations through trivial inspection. Many instances can be created for each object and they can be 'brought to life' to interact with one another within the CrabWorld depicted in Figure 7.

Figure 7:
Lobster, Crab, Worm Scenario (Source code: Kölling, 2010)



The learner is submerged (embodied) into this CrabWorld and faced by many of the following unsolved questions that need a solution or an algorithm:

- i. How can one ensure natural movement by utilising more than one image of a crab to simulate leg movement? This is not always picked up by every learner, but learners' attention to detail is sharpened through looking at other scenarios within the LMS and applying a solution to this formulated problem. In this instance, the teacher needs to formulate the problem to make the task more digestible. Figure 8 illustrates the coding of two images that represents one crab:

Figure 8:
Greenfoot coding of Crab movement



(Source code: Kölling, 2010)

- ii. What would the CrabWorld look like within the virtual world of the Greenfoot class? This requires the learner to understand the dimensions of the x and y axis and calculate the relative position of the animal considering the width and height of the Greenfoot world. Embodied cognition is required by the learner as the animal continuously moves and turns away from the edges. Although this implies intense mathematical thinking, the aim is on forcing the user to produce a solution to the formulated problem in the form of an algorithm and computational steps that will comply with the properties of the computational model.

- iii. How can we ensure that each animal will abide by its eating habits, i.e. that the lobster will eat the crab and that the crab may eat worms, and so on? Here, CT is enforced by considering the specific computational model, which is an object-messaging model. The learner abstracted/encapsulated the code into a method `lookForWorm()` in Figure 8. The detail is hidden within `lookForWorm()` method. The instances of these animals can be created and each instance will act independently through messaging.
- iv. Which methods are common to all the animals that should be housed in an `Animal` class as depicted in Figure 7? This forms a super class (generalisation) from which all animals inherit certain common methods, i.e. methods used by all the creatures or animals. All animals must move and sense the edge of the world. Once again, the detailed code to accomplish this will be embedded in methods that will perform these actions. The learners identify such methods through mental mechanisms in APOS mental structures. The concept of generalisation is a mental mechanism within reflective abstraction and forms part of CT. It can be seen as decomposing the classes of animals and identifying those generalised methods that link directly with CT (Selby & Woollard, 2014). Not only is this aligned with APOS mental mechanisms within reflective abstraction such as encapsulation and de-encapsulation, but it also forms part of CT. These common methods are created using the Greenfoot Language computational notation.
- How do the animals behave when they invade one another's space?
 - What sounds do they make if one animal devours the other?
 - What happens to the object that is devoured?

The learners are submerged in CT to find solutions for these formulated problems and translate the problems into code.

- v. The learner needs to understand the questions and asks for help if he/she does not understand. Group work was used to brainstorm the questions. Each learner was given the opportunity to create an algorithm for the task at hand. The beauty of Greenfoot is that the outcome explains the understanding of the learner, since the scenario can be run as a whole or one step at a time and is visible to both learner and teacher. Based on the outcome, the learner can then decompose or reverse the computations. Techniques such as refactoring of code, encapsulation of code into methods, and creating classes and objects to solve the problem are investigated and 'drilled in'. Drilling is a term used during language learning that coincides with the APOS term process where the learner moves on from simply taking Actions to memorising these actions as a Process to form part of a schema. It can be described as scaffolding where learners develop more schemas as they apply CT and reflective abstraction. Positive results were noticed with larger projects and fewer teacher interventions. Learners formulated their own problems and each arrived at solutions, different in code, but with similar outcomes.

5. FINDINGS

The findings were accomplished through the following iterative interventions:

Intervention 1: Abstraction

Abstraction as a pillar of computational thinking was investigated by learners through videos, lectures and assessments. The APOS framework was then introduced through Greenfoot PL as a new technology and approach or PL belief.

Intervention 2: Actions and processes through two steps

Step 1: Moodle and Greenfoot

Alignment and investigation of actions and processes using a PL and LMS were accomplished through lectures, videos and assessments.

Step 2: Coding

Code within Greenfoot was composed with the help of lectures, videos and Moodle, and assessed as small projects or assignments.

Intervention 3: Encapsulation of code

Code was rolled out through lectures and videos, and assessed by means of practical class tests. The technical skills on encapsulation through the refactoring of code spearheaded an object and schema creation within APOS. The rollout was then tested through the implementation of small programs and assessments.

Intervention 4: Object and schema

Code was rolled out through lectures and videos, and assessed by means of practical class tests. Using encapsulation within Greenfoot, the concept of objects and schemas could be rendered within the thinking processes of the learner, which illustrated the last stages of APOS. The technical skills, objects and schemas were then tested through implementation of a project.

These interventions were always linked to a mathematics problem so the learners can identify the connection of APOS theory across domains.

5.1 Basic findings derived from these interventions include:

- i. It is difficult, if not impossible, to change learners' viewpoint on mathematics through mathematics itself due to mathematical beliefs.
- ii. A PL such as Greenfoot does fit the APOS model.
- iii. The learners' approach to mathematical thinking through the APOS framework can be aligned using a programming language to enforce mathematical thinking.
- iv. The involvement of the headmaster or line manager is paramount in the rollout of the research.

6. DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

The low pass rates of learners taking mathematics and science at high-school level are of great concern. This paper proposes a conceptual framework that can be used to improve the CT of learners using a PL and LMS in applying APOS theory. It is well established that the APOS theory positively contributes to the success rates of learners in mathematics (Arnon et al., 2014).

This paper does not focus on specific subjects *per se*, but rather encourages the development and improvement of CT among all learners. The learner's mind should not be occupied by mathematics and science. The pre-set idea that only 'clever or mathematics inclined learners' are able to take programming, was removed, as pointed out by Papert (2005) with his Pop-Ed theories.

In many cases, learners are exposed to the monotonous process of working through existing mathematics papers. This activity takes the learner down the road of storing image snapshots of a mathematical concept, which is called rote learning. Learners should build schemas on mathematical concepts. Students

accepted by universities, based on the assurance of being logical thinkers, built on snapshot techniques of mathematical concepts, may not cope where a higher cognitive level of formal operations is expected.

After using the proposed framework, interviews were conducted, where a learner (X) mentioned that during group work some so-called 'superior' mathematical learners struggled in coming up with solutions on formulated problems. Learner (X) suggested that she should do better in mathematics than the so-called 'superior' learners. Such special moments may motivate those to reflect on their performance.

In conclusion, APOS theory supported by a PL and LMS enhances the learner's CT ability. By applying this framework, learners will be able to find solutions through reflective abstraction in actions, processes, objects and schemas, which support the arguments and findings of Dubinsky (1991) and Arnon et al. (2014) in mathematics.

7. FURTHER RESEARCH

Further research needs to be done on the adoption of the innovation when conducting local and broad-impact evaluations as discussed by Bannan (2013). This requires a positivist approach within a developmentalist paradigm (Weber, 2010). Acceptability will then occur outside the interpretivist approach. It is still unclear what role the teacher plays in terms of understanding the APOS theory, PL and LMSs due to APOS being unfamiliar to the majority of mathematics teachers in RSA.

As this study only focused on one private school, it is recommended that more schools, including state schools, get involved. Preferably a larger number of learners starting in Grade 8 should be researched using a PL and APOS theory as lens.

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