Regression-SARIMA modelling of daily peak electricity demand in South Africa

Delson Chikobvu
Department of Mathematical Statistics and Actuarial Science, University of the Free State, South Africa

Caston Sigauke
Department of Statistics and Operations Research, School of Mathematical and Computer Sciences, University of Limpopo, South Africa

Abstract
In this paper, seasonal autoregressive integrated moving average (SARIMA) and regression with SARIMA errors (regression-SARIMA) models are developed to predict daily peak electricity demand in South Africa using data for the period 1996 to 2009. The performance of the developed models is evaluated by comparing them with Winter’s triple exponential smoothing model. Empirical results from the study show that the SARIMA model produces more accurate short-term forecasts. The regression-SARIMA modelling framework captures important drivers of electricity demand. These results are important to decision makers, load forecasters and systems operators in load flow analysis and scheduling of electricity.

Keywords: daily peak demand; SARIMA; regression-SARIMA; short term load forecasting

1. Introduction
Modelling daily peak electricity demand is important as it provides short-term forecasts which will assist system operators in dispatching of electrical energy. Prediction of load demands is very important for decision making processes in the electricity sector. Decision making in this sector involves planning under uncertainty. This involves, for example, finding the optimal day to day operation of a power plant and even strategic planning for capacity expansion. The demand of electricity forms the basis for power system planning, power security and supply reliability (Ismail et al., 2009). It is important therefore, to produce very accurate forecasts as the consequences of underestimation or overestimation can be costly. Underestimation has a serious negative impact on the national electricity supply system of a country. It may result in the national electricity supply system becoming unstable thus leading to supply interruption if left unchecked. This may even lead to further loss of business as restoration of a plant or construction of new plants takes a long time before generation can start. If the entire national electricity supply system were to shut down, it would take days, possibly even weeks to restore. Overestimation results in wastage of resources due to excess production. As noted by Taylor (2008), accurate short-term forecasts are needed by both generators and retailers of electricity particularly, during periods of abnormal peak load demand. Accurate forecasts will enable effective load shifting between transmission substations. In order to improve forecast accuracy it is important to combine statistical forecasting methods together with judgmental techniques. Through experience, the judgmental experts develop intuitive relationships between electrical load and weather parameters, time of day, day of week, season and time lag of response, (Ismail et al., 2009).

On the other hand, statistical techniques provide a scientific approach for producing consistent and accurate forecasts.

Load forecasting has been studied extensively for over four decades using classical time series, regression and neural network methods. Amaral et al. (2008) developed a smooth transition periodic autoregressive (STPAR) model and this was evaluated against alternative load forecasting models using electricity load series data from Australia. STPAR proved to be a useful tool when forecasting...
the electricity load. In their paper, Sumer et al. (2009) developed ARIMA, SARIMA (seasonal ARIMA) and regression models with seasonal latent variable in forecasting electricity demand of the data from ‘Kayseri and Vicinity Electricity Joint-Stock Company’. Their results show that the regression model with seasonal latent variable is more efficient than ARIMA and SARIMA. Taylor (2008) used minute-by-minute data on British electricity demand to evaluate 10-30 minutes ahead prediction methods. It is argued that such short lead times are important for the real-time scheduling of electricity generation. ARIMA models, an adaptation of the Holt-Winters’ exponential smoothing model and an exponential smoothing method that focuses on the evolution of intraday cycle, are used. Out of these methods, the double seasonal adaptation of the Holt-Winters’ exponential smoothing model gives the best results and this is consistent with results from previous studies. Soares and Medeiros (2008) consider a two-level method for hourly electricity load. A Two-Level Seasonal Autoregressive (TLSAR) model is developed and compared with a modified version of a SARIMA model called Dummy-Adjusted SARIMA (DASARIMA). A specific class of seasonal ARIMA models (the benchmark model) and the generalized long memory (GLM) model discussed by Soares and Souza (2006) are better than DASARIMA. A possible extension of this methodology to combining forecasts, interval forecasts and forecast density evaluation is suggested.

Use of regression based methods and neural network models are also discussed in the literature. A hybrid neural network model for daily electrical peak load forecasting (PLF) is presented by Amin-Naseri and Soroush (2008). A novel approach for clustering data by using a self-organizing map is proposed, for which a feed forward neural network (FFNN) is developed for each cluster to provide the PLF. It is concluded that the proposed hybrid model produces superior forecasts than those of the linear regression. The modelling and short-term forecast of daily power demand in the state of Victoria, Australia, is discussed in Truong et al. (2008). A two-dimensional wavelet base state dependent (SDP) modelling approach is adopted to formulate a compact mathematical model that is used to forecast daily peak power demand from 9 to 24 August 2007. With a MAPE of 1.9%, the model is found to be effective. A non-linear multivariable regression model for mid-term energy forecasting of power systems in annual time base is developed by Tsekouras et al. (2007) and applied to the Greek power system using different categories of low voltage customers. The model includes a correlation analysis of the selected input variables and performed an extensive search to select the most appropriate variables. Ismail et al. (2009) use a rule-based forecasting approach for forecasting peak load electricity demand. The authors conclude that rule-based forecasting increases the forecast accuracy when compared to the traditional SARIMA model and that improvement depends on the conditions of the data, knowledge development and validation. Ramanathan et al. (1997) develop a simple and flexible set of models for hourly load forecasting and probabilistic forecasting. These are multiple regression and exponential smoothing methods. The models developed perform well against a wide range of alternative models.

Some research has been done on South African electric load data. Notable contributions in this area are those of Amusa et al. (2009) who apply the bounds testing approach to co integration within an autoregressive distributed lag framework to examine the aggregate demand for electricity in South Africa during the period 1960-2007.

Hahn et al. (2009) gave an overview of some of the methods used in demand load forecasting. The methods were classified into regression based, time series, state space and kalman-filtering. Artificial and computational intelligence methods are also suggested. Neural networks and support vector regression methods fall into this class. However, the current trend is to develop hybrid models as they are seen to be more robust. A recent review forecasting is given in Munoz et al. (2010) and Suganthi and Samuel (2011).

The paper discusses the application of the SARIMA and regression with SARIMA errors (regression-SARIMA) to daily peak demand (DPD) forecasting in South Africa. The regression-SARIMA modelling framework captures important drivers of electricity demand. These factors are weather variables, economic and calendar effects and are known to influence electricity demand. An extension of the regression-SARIMA modelling framework is discussed in detail in Sigauke and Chikobvu (2011). The rest of the paper is organized as follows. A detailed discussion of the data including fitting a probability distribution is presented in Section 2. SARIMA and regression-SARIMA models are then developed in Section 3. Empirical results from the study are covered in Section 4. A comparative analysis of the two models together with the Winter’s triple exponential smoothing model is discussed in section 5. The summary and conclusion of the paper are covered in section 6.

2. Data

It is important that the amount of electricity drawn from the grid and the amount generated balances (Cottet and Smith, 2003; Taylor, 2006) and this amount is called electricity load which is equal to electricity demand in the absence of blackouts and load-shedding. Aggregated DPD data from all sectors of the South African economy for the period January 1996 to December 2009 is used in this
paper. The data is from Eskom, South Africa’s power utility company. DPD is the maximum hourly demand in a 24-hour period. There are 5097 observations. Table 1 gives a summary of the descriptive statistics of DPD.

The time series plot of DPD in Figure 1 shows a positive linear trend and strong seasonality. The null hypothesis of a stochastic trend is accepted under the Augmented-Dickey Fuller unit root test.

A spectral analysis is carried out to investigate the periodicity in the data. The spectral density shows a seven day periodicity. We fit a probability distribution to the sample data. Table 2 shows a comparison of alternative distributions fitted. The normal distribution is the best fitting distribution since it has the largest log likelihood and smallest Akaike information criterion statistics. The estimated parameters of the normal distribution together with the standard errors in parentheses are: mean = 27406.05 (53.23) MW and standard deviation = 3808.76 (37.69) MW. Empirical and theoretical cumulative distribution functions (CDFs) for the Weibull, normal and lognormal distributions given in Figure 2 also show that the normal distribution is the best fitting distribution.

The probability density function of DPD was estimated using kernel density estimation (Silverman, 1986) and is plotted in Figure 3.

### 3. The models

#### 3.1 SARIMA Model

The general SARIMA model can be represented analytically as:

\[
\phi(B) \Phi(B^s) \nabla^d \nabla_p^s z_t = \theta(B) \Theta(B^s) a_t, \quad a_t \sim N(0, \sigma^2)
\]

where \( z_t \) represents DPD at time \( t \), \( a_t \sim N(0, \sigma^2) \) is the error term at time \( t \), \( s \) is the seasonal length, \( B \) is a backshift operator \( (B z_t = z_{t-1}) \). \( \phi(B) = (1 - \phi_1 B - \cdots - \phi_p B^p) \) is the nonseasonal autoregressive (AR) operator, \( \Phi(B^s) = (1 - \Phi_1 B^s - \cdots - \Phi_p B^{ps}) \) is the seasonal AR operator, \( \theta(B) = \cdots \cdots \)

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### Table 1: Descriptive statistics for DPD

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
<th>Std Dev</th>
<th>Skew</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPD</td>
<td>27406</td>
<td>27289</td>
<td>37158</td>
<td>16601</td>
<td>3809</td>
<td>0.0703</td>
<td>2.2710</td>
</tr>
</tbody>
</table>

### Table 2: Comparison of alternative distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Log Likelihood</th>
<th>AIC</th>
<th>Estimated parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>-49257.4</td>
<td>98518.8</td>
<td>Mean = 27406.05 (53.23) Std dev = 3808.76 (37.69)</td>
</tr>
<tr>
<td>Lognormal</td>
<td>-49275.28</td>
<td>98554.55</td>
<td>Meanlog = 10.2087 (0.001972) Sdlog = 0.1408 (0.001394)</td>
</tr>
<tr>
<td>Weibull</td>
<td>-49368.46</td>
<td>98740.93</td>
<td>Shape = 7.9097 (0.0844) Scale = 29074.87 (54.399)</td>
</tr>
</tbody>
</table>

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![Figure 1: Time series plot of daily peak electricity demand (01-01-1996 to 14-12-2009)](image1)

![Figure 2: Empirical and theoretical CDFs for the Weibull, normal and lognormal distributions](image2)
3.2 Regression-SARIMA Model

Regression-SARIMA models are multivariate linear regression models which work well when the relationship between demand and the predictor variables is linear. Electricity demand is generally divided into short-term, medium-term and long-term forecasting. The regression-SARIMA model is one in which the mean function of the time series is described by a linear combination of regressors. The covariance structure of the series is that of the SARIMA process. The regression-SARIMA model reduces to a SARIMA model if the regressors are not used. The regression-SARIMA model captures important drivers of electricity demand such as calendar, weather and economic factors.

The paper concentrates on short-term forecasting and the inclusion of calendar effects in the modelling framework. Weather variables such as temperature are not included in this study. A detailed discussion of the influence of temperature on electricity demand in South Africa is given in (Sigauke and Chikobvu, 2010). Several papers in literature have adopted the same strategy of not including temperature (Carpinteiro et al., 2004; Taylor et al., 2006; Soares and Souza, 2006; Soares and Medeiros, 2008). The regression-SARIMA model used in this study is given as:

\[
(1 - \theta_1 B - \cdots - \theta_q B^q) \phi(B) \theta(B^d) \theta(B^p) \beta(B^d) \beta(B^p) y_t = \phi(B) \theta(B^d) \theta(B^p) \beta(B^d) \beta(B^p) e_t
\]

\[
\gamma \left[ \delta_{t - \sum_{r=1}^{12} \lambda_r m_t - \sum_{r=1}^7 \tau_r g_r} - \gamma H_{j-1} - \mu H_j - \rho H_{j+1} \right]
\]

4. The empirical results

In order to have a better understanding of the daily demand patterns of electricity we calculate the daily seasonal indices. A summary of these daily indices is given in Table 3. Day 7, which is Sunday, had the lowest seasonal index of 93.265% showing that, on average, the consumption of electricity is 6.735% below average consumption. The highest index of 103.078% on day 3, which is Wednesday, indicates that there is an above average consumption of 3.078%. For the rest of the week days the load variations are small (see Figure 4).

<table>
<thead>
<tr>
<th>Season</th>
<th>Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>102.343</td>
</tr>
<tr>
<td>Tuesday</td>
<td>103.006</td>
</tr>
<tr>
<td>Wednesday</td>
<td>103.078</td>
</tr>
<tr>
<td>Thursday</td>
<td>102.915</td>
</tr>
<tr>
<td>Friday</td>
<td>100.224</td>
</tr>
<tr>
<td>Saturday</td>
<td>95.1683</td>
</tr>
<tr>
<td>Sunday</td>
<td>93.265</td>
</tr>
</tbody>
</table>

4.1 SARIMA model results

A summary of the estimates of the parameters of the best fitting model together with some important statistics are given in Table 4. The data was transformed using seasonal differencing and also by taking natural logarithms. The transformed data was also found to follow a normal distribution.
The model can be written as:

\[ (1 - \varphi_1 B - \varphi_2 B^2 - \varphi_3 B^3 - \varphi_4 B^4) (1 - B^7) \ln z_i = (1 - \theta_1 B - \theta_2 B^2) (1 - \Theta_1 B^7 - \Theta_2 B^{14} - \Theta_3 B^{21}) \alpha_i \]

and substituting the values of the parameters we get:

\[ (1 - 0.84 B - 0.39 B^2 + 0.26 B^4 - 0.03 B^6) (1 - B^7) \ln z_i = (1 + 0.43 B^2 + 0.24 B^3) (1 + 0.81 B^7 + 0.12 B^{14} + 0.05 B^{21}) \alpha_i \]

Several SARIMA models are considered and the best model has a root mean square error (RMSE) of 544.79, mean absolute error (MAE) of 370.81 and a mean absolute percentage error (MAPE) of 1.39%.

### 4.2 Regression-SARIMA model results

Table 5 shows a summary of the estimates of the variables of the regression-SARIMA model together with the p-values in parentheses. A day before a holiday has a negative coefficient indicating a reduction in electricity consumed. There is a significant reduction in electricity demand during holidays as evidenced by the coefficient of the dummy variable \(H_j\) in table 5.

After substituting the values of the parameters into the developed regression-SARIMA model, we get:

\[ (1 - 1.65 B + 0.73 B^2 - 0.17 B^3 + 0.17 B^4 - 0.07 B^5) (1 - 0.69 B^7)(1 - B^7) \]

\[ (\gamma_i + 157 \text{July} - 159 \text{October} + 25 \text{Tuesday} + 28 \text{Wednesday} + 25 \text{Thursday} + 25 \text{Friday} + 42 \text{Saturday} + 30 \text{Sunday} + 388 H_j + 117 H_{j+1}) = (1 + 0.82 B)(1 + 1.45 B^7 - 0.48 B^{14} + 0.02 B^{21}) \alpha_i + 54 \]

After considering several regression-SARIMA models, the best model has a RMSE of 539.27, MAE of 381.17 and a MAPE of 1.427%.
5. Comparative analysis

The paper concentrated on daily peak demand forecasting, which is important for providing short term forecasts which will assist in optimal dispatching of electrical energy. The mean absolute percentage error (MAPE) and the root mean square error (RMSE) are used for comparing the models in short-term demand forecasting up to seven days ahead. The training period was 1 January 1996 to 30 October 2009. The performance of the developed models is evaluated by comparing them with Winter’s triple exponential smoothing model with $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 0.1$. Table 6 shows a comparative analysis of the SARIMA and regression-SARIMA models, together with results from using Winter’s triple exponential smoothing model.

The SARIMA model has the least MAPE, showing that it is the best fitting model.

The graphical plot of the out of sample forecasts using the SARIMA model, approximate 95% prediction intervals and actual daily peak for the first 14 days of November 2009, are given in Figure 5. LCL represents the lower 95% confidence interval while UCL represents the 95% upper confidence interval. The actual peak demand falls within the prediction interval for all the 14 days. The SARIMA model seems to be useful for making short-term forecasts of daily peak demand. The probability density of the forecasted values for the first fourteen days of November 2009 is shown in Figure 6. The density shows the full probability distribution of the possible future values of peak demand over the 14 day period. The density is bimodal. This is important for load forecasters and systems operators in load flow analysis and dispatching of electrical energy.

6. Conclusion

In this paper, a time series methodology is presented to forecast DPD for Eskom using SARIMA and

![Figure 5: Graphical plot of the forecasts, actual peak demand with residuals and 95% confidence limits. The x-axis represents the date and the y-axis is demand in megawatts]

![Figure 6: Probability density of the forecasted values for the first 14 days of November 2009. The x-axis represents daily peak electricity demand in megawatts]
regression-SARIMA models. The regression-SARIMA model captures important drivers of electricity demand. Empirical results from the study show that the SARIMA model produces more accurate short-term forecasts. The regression-SARIMA model can be improved if weather variables such as temperature are included and also grouping holidays according to their load reduction patterns. The regression-SARIMA model is simple to implement, reliable and provides information about the importance of each predictor variable. The results from using a regression-SARIMA model are relatively robust. Another interesting area for further study would be density forecasting of daily peak electricity demand in which density forecasts provide the estimates of full probability distributions of the possible future values of demand. These areas will be studied elsewhere.

Acknowledgments
The authors are grateful to the power utility company of South Africa, Eskom for providing the data, the University of Limpopo and University of the Free State for using their resources and to the numerous people who assisted in making comments on this paper.

Appendix

A general multiplicative SARIMA model for the DPD time series $y_t$ can be written as:

$$
\phi(B)\phi(B^s)\nabla^d\nabla^p_s y_t = \\
\theta(B)\theta(B^s)a_t,
$$

where $a_t \sim N(0, \sigma^2)$.

An extension of the SARIMA model in equation (6) involves the use of a time varying mean function which we model through linear regression effects. A linear regression model which can be used to extend the time varying mean function is given as:

$$
y_t = \sum_{f=1}^k \beta_f x_{ft} + z_t
$$

where $y_t$ is the dependent time series, $x_{ft}$ are the explanatory variables, $\beta_f$ are the regression parameters and $z_t = y_t - \sum_{f=1}^k \beta_f x_{ft}$.

We then substitute the expression for $z_t$ in equation (6) to get

$$
\phi(B)\phi(B^s)\nabla^d\nabla^p_s y_t = \\
\theta(B)\theta(B^s)a_t
$$

(9)

where

$$
\nabla^d\nabla^p_s = (1 - B)^d(1 - B^s)^p
$$

In order to capture the day of the week effect dummy variables are introduced and defined as follows:

$$
g_r = \begin{cases} 
1, & r = \text{Mon,...,Sun} \\
0, & \text{otherwise} 
\end{cases}
$$

(13)

The daily peak demand decreases during holidays. Some companies do close earlier on a day before a holiday. There is a reduction in electricity demand a day before and after a holiday (Ismail et al., 2008). To take into account the effects of holidays the following dummy variables, $H_j$, $H_{j-1}$ and $H_{j+1}$ are introduced

$$
H_j = \begin{cases} 
1, & j = \text{holiday} \\
0, & \text{otherwise} 
\end{cases}
$$

(14)

$$
H_{j-1} = \begin{cases} 
1, & j = \text{day before holiday} \\
0, & \text{otherwise} 
\end{cases}
$$

(15)

$$
H_{j+1} = \begin{cases} 
1, & j = \text{day after holiday} \\
0, & \text{otherwise} 
\end{cases}
$$

(16)

If a holiday falls on a Sunday, the following Monday is declared a public holiday. To take into account the monthly seasonality effect a dummy variable $m_i$ is introduced.

$$
m_i = \begin{cases} 
1, & i = \text{Jan,...,Dec} \\
0, & \text{otherwise} 
\end{cases}
$$

(17)

Our regression-SARIMA model is then written as:

$$
\phi(B)\phi(B^s)\nabla^d\nabla^p_s [y_t - \sum_{i=1}^L \sum_{r=1}^K \sum_{t=1}^T D_r] \\
\gamma H_{j-1} - \mu H_j - \rho H_{j+1}
$$

$$
= \theta(B)\theta(B^s)a_t
$$

where

$$
\nabla^d\nabla^p_s = (1 - B)^d(1 - B^s)^p
$$
References

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